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On type IIA Penrose limit and $\mathcal{N} = 6$ Chern-Simons theories

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Abstract: Recently, Aharony, Bergman, Jafferis and Maldacena proposed that the $\mathcal{N} = 6$ Chern-Simons gauge theories are holographically dual to the M-theory backgrounds with multiple M2-branes on orbifolds C^4/Z_k. When k is large, they have the type IIA string description. In this paper we analyze the Penrose limit of this IIA background and express the string spectrum as the conformal dimensions of operators in the gauge theories. For BPS operators, we can confirm the agreements between the IIA string on plane waves and the gauge theories. We point out that there exist BMN-like operators in the gauge theories, though their holographic interpretation does not seem to be simple. Also we analyze the weak coupling limit of this theory and show that the Hagedorn/deconfinement transition occurs as expected.

Keywords: Penrose limit and pp-wave background, AdS-CFT Correspondence, M-Theory.

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Contents

1. Introduction

Recently, there have been very interesting progresses on the world-volume theory of multiple M2-branes in M-theory. Since we expect that this theory is dual to $AdS_4 \times S^7$ in the decoupling limit, it should be described by a three dimensional $\mathcal{N} = 8$ superconformal theory. The superconformal Chern-Simons theories have been constructed in [1−3], though they are less supersymmetric. Bagger, Lambert and Gustavsson constructed the three dimensional $\mathcal{N} = 8$ supersymmetric theory based on the Lie 3-algebra structure [4, 5]. For subsequent developments refer to [6]–[44]. If we assume the positive metric of the Lie 3-algebra, the algebra constraints the Lagrangian strongly [14, 18−20]. This only allows us to construct a $\mathcal{N} = 8$ supersymmetric theory which is dual to two M2-branes. If we allow the non-degenerate metric, we can find $\mathcal{N} = 8$ supersymmetric theories where we can take the number of branes N arbitrary large [22−24, 34, 35, 38, 41]. However, this theory can be reduced to the well-known $\mathcal{N} = 8$ super Yang-Mills theory on N $D2$-branes.

Aharony, Bergman, Jafferis and Maldacena have constructed very interesting $\mathcal{N} = 6$ $\mathrm{U}(N)\times\mathrm{U}(N)$ Chern-Simons theories (ABJM theory) and proposed that they are dual to the world-volume theory of N M2-branes for the arbitrary number of N [45] (see also [46, 47] for further study). This theory is parameterized by the level k of the Chern-Simons gauge theory. The ABJM theory with level k and the gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ is argued to describe the world-volume theory of M2-branes on the orbifold C^4/Z_k. When the gauge group is $\mathrm{SU}(2) \times \mathrm{SU}(2)$, this theory becomes equivalent to the BLG theory [4, 5].

In the paper [45], it was also pointed out that the ABJM theory at large k is dual to the type IIA string on $AdS_4 \times CP^3$. This offers us to study a new AdS_4/CFT_3 duality where both AdS and CFT side are tractable with the present knowledge of string theory. The CFT side is defined by a 't Hooft limit $N \to \infty$ of ABJM theory with $\frac{N}{k}$ kept finite.

As a next step, it will be very intriguing to check this proposed duality from both the IIA string theory and the Chern-Simons gauge theory side. In this paper we would like to report a modest progress in this direction. Namely, we would like to consider the Penrose limit of the type IIA background because in this limit the string theory becomes solvable even in the presence of α' corrections and RR-fluxes. It has been well-known that the Penrose limit of type IIB string on $AdS_5 \times S^5$ successfully reproduces the results of BMN operators in the $\mathcal{N} = 4$ super Yang-Mills theory [48].

We will show that the Penrose limit of this IIA string background becomes the plane wave background with 24 supersymmetries studied in [49] after an appropriate coordinate transformation. We find[1] the exact string spectrum and express the results as the anomalous dimensions of operators in the ABJM theory. We will also notice that in the ABJM theory, we can define a BMN-like operator and we will compute its anomalous dimension to leading order of the effective 't Hooft coupling.

We will also study the weak coupling limit $k \to \infty$ of the ABJM theory. Since we can neglect the non-singlet flux contributions in this limit, we can analyze the partition function of the ABJM theory compactified on $S^1 \times S^2$ analytically and show that the deconfinement/confinement transition occurs at a specific temperature as expected from the Hagedorn transition in the string theory side.

This paper is organized as follows. In section 2, we review and analyze in the detail the reduction of $AdS_4 \times S^7$ background in M-theory to the type IIA string. We will also compute the holographic entanglement entropy of the ABJM theory. In section 3, we take the Penrose limit of the IIA background. We compute the string spectrum and express the results from the gauge theoretic viewpoint. In section 4, we define BMN-like operators in ABJM theory and compute their anomalous dimensions. In section 5, we evaluate the partition of free ABJM theory and confirm the Hagedorn transition at a specific temperature. In section 6 we summarize our conclusions.

After we finished this paper, we noticed a very interesting preprint [47] by Bhattacharya and Minwalla, where an agreement of the supersymmetric index between the $\mathcal{N} = 6$ Chern-Simons theory and its dual supergravity was shown. The section 5 of our paper has some overlap with their calculations.

[1]The same plane-wave also appears in the study of the gravity dual of a 2+1 super Yang-Mills with SU(2|4) symmetry [50], where the string spectrum is compared with the Yang-Mills operators.

2. M2-branes on C^4/Z_k and reduction to IIA string

2.1 M2-brane solution

We start with the 11 dimensional supergravity action

$$S = \frac{1}{2\kappa_{11}^2}\int dx^{11}\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right) - \frac{1}{12\kappa_{11}^2}\int C^{(3)}\wedge F^{(4)}\wedge F^{(4)}, \tag{2.1}$$

where $\kappa_{11}^2 = 2^7\pi^8 l_p^9$. The equations of motions read

$$R^\mu_\nu = \frac{1}{2}\left(\frac{1}{3!}F^{\mu\alpha\beta\gamma}F_{\nu\alpha\beta\gamma} - \frac{1}{3\cdot 4!}\delta^\mu_\nu F_{\alpha\beta\rho\sigma}F^{\alpha\beta\rho\sigma}\right), \tag{2.2}$$

and

$$\partial_\sigma(\sqrt{-g}F^{\sigma\mu\nu\xi}) = \frac{1}{2\cdot(4!)^2}\epsilon^{\mu\nu\xi\alpha_1\cdots\alpha_8}F_{\alpha_1\cdots\alpha_4}F_{\alpha_5\cdots\alpha_8}. \tag{2.3}$$

Then the near horizon limit of M2-brane solution becomes $AdS_4\times S^7$

$$ds^2 = \frac{R^2}{4}ds^2_{AdS_4} + R^2 d\Omega_7^2, \tag{2.4}$$

where the radius R is given by $R = l_p(2^5N'\pi^2)^{\frac{1}{6}}$ (N' is the number of the M2-branes). The four form flux is found to be

$$F^{(4)} = \frac{3R^3}{8}\epsilon_{AdS_4}, \tag{2.5}$$

where ϵ_{AdS_4} is the unit volume form of the AdS_4 space. If we assume the Poincare metric $ds^2_{AdS_4} = \frac{dr^2}{r^2} + r^2\sum_{\mu=0}^2 dx^\mu dx_\mu$, we have $\epsilon_{AdS_4} = r^2$ or equally $F_{012r} = \frac{3R^3r^2}{8}$.

2.2 The reduction to IIA

We take the Z_k orbifold of S^7 and reduce the M-theory background $AdS_4\times S^7/Z_k$ to the type IIA string background following [45]. We can express S^7 by the complex coordinate X_1, X_2, X_3 and X_4 with the constraint $|X_1|^2+|X_2|^2+|X_3|^2+|X_4|^2 = 1$. We can parameterize S^7 by

$$\begin{aligned}
X_1 &= \cos\xi\ \cos\frac{\theta_1}{2}\ e^{i\frac{\chi_1+\varphi_1}{2}},\\
X_2 &= \cos\xi\ \sin\frac{\theta_1}{2}\ e^{i\frac{\chi_1-\varphi_1}{2}},\\
X_3 &= \sin\xi\ \cos\frac{\theta_2}{2}\ e^{i\frac{\chi_2+\varphi_2}{2}},\\
X_4 &= \sin\xi\ \sin\frac{\theta_2}{2}\ e^{i\frac{\chi_2-\varphi_2}{2}},
\end{aligned} \tag{2.6}$$

where the angular valuables run the values $0\le\xi<\frac{\pi}{2}$, $0\le\chi_i<4\pi$, $0\le\varphi_i\le 2\pi$ and $0\le\theta_i<\pi$. Then the metric of S^7 can be written as

$$\begin{aligned}
ds^2_{S^7} = d\xi^2 &+ \frac{\cos^2\xi}{4}\left[(d\chi_1+\cos\theta_1 d\varphi_1)^2 + d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2\right]\\
&+\frac{\sin^2\xi}{4}\left[(d\chi_2+\cos\theta_2 d\varphi_2)^2 + d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2\right].
\end{aligned} \tag{2.7}$$

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Now we define new coordinates

$$\chi_1 = 2y + \psi, \qquad \chi_2 = 2y - \psi. \tag{2.8}$$

The Z_k orbifold action is now given by $y \sim y + \frac{2\pi}{k}$. Then the metric of S^7 can be rewritten as follows

$$ds^2_{S^7} = ds^2_{CP^3} + (dy + A)^2, \tag{2.9}$$

where

$$A = \frac{1}{2}(\cos^2\xi - \sin^2\xi)d\psi + \frac{1}{2}\cos^2\xi\cos\theta_1 d\varphi_1 + \frac{1}{2}\sin^2\xi\cos\theta_2 d\varphi_2, \tag{2.10}$$

and

$$\begin{aligned} ds^2_{CP^3} =& \, d\xi^2 + \cos\xi^2\sin^2\xi\left(d\psi + \frac{\cos\theta_1}{2}d\varphi_1 - \frac{\cos\theta_2}{2}d\varphi_2\right)^2 \\ &+ \frac{1}{4}\cos^2\xi\left(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2\right) + \frac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2). \end{aligned} \tag{2.11}$$

This expression (2.11) of CP^3 can be found in e.g. [51].

By comparing the above result with the conventional reduction formula (below we always work with the string frame metric setting $\alpha' = 1$)

$$ds^2_{11D} = e^{-2\phi/3}ds^2_{\mathrm{IIA}} + e^{\frac{4}{3}\phi}(d\tilde{y} + \tilde{A})^2, \tag{2.12}$$

where $\tilde{y}$ is compactified as $\tilde{y} \sim \tilde{y} + 2\pi$. Since we are taking the Z_k orbifold, we identify $\tilde{y} = ky$, which leads to the value of dilaton

$$e^{2\phi} = \frac{R^3}{k^3} = 2^{\frac{5}{2}}\pi\sqrt{\frac{N}{k^5}}. \tag{2.13}$$

The RR 2-form $F^{(2)} = d\tilde{A}$ in the type IIA string is explicitly given by

$$\begin{aligned} F^{(2)} =& \, k\Big(-\cos\xi\sin\xi d\xi \wedge (2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2) \\ &-\frac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1 \wedge d\varphi_1 - \frac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2 \wedge d\varphi_2\Big), \end{aligned} \tag{2.14}$$

while the RR 4-form remains the same

$$F^{(4)} = \frac{3R^3}{8}\epsilon_{AdS_4}. \tag{2.15}$$

The string frame metric now becomes

$$ds^2_{\mathrm{IIA}} = \tilde{R}^2(ds^2_{AdS4} + 4ds^2_{CP3}), \tag{2.16}$$

where $\tilde{R}^2 = \frac{R^3}{4k} = \pi\sqrt{\frac{2N}{k}}$. In this way we obtain the $AdS_4 \times CP^3$ IIA background [45, 52]. This background preserves the 24 supersymmetries including the near horizon enhancement as it is dual to three dimensional $\mathcal{N} = 6$ superconformal symmetry.

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2.3 Holographic entanglement entropy

To measure the degrees of freedom in a given conformal field theory, a useful quantity is known as the entanglement entropy S_A in addition to the ordinary thermodynamical entropy. We expect that it becomes more important in CFT_3 since in odd dimensions we do not have a precise definition of the central charges. We trace out the subsystem A which is defined by an infinite strip with the width l. Then the holographic area law formula in [53] leads to the following result[2] from the analysis of minimal surfaces in the Poincare AdS_4

$$S_A = \frac{\sqrt{2}}{6\pi} N^2 \sqrt{\frac{k}{N}} \left(\frac{L}{a} - 2\pi \frac{\Gamma(3/4)^2}{\Gamma(1/4)^2} \cdot \frac{L}{l} \right), \tag{2.17}$$

where L represents the infinitely large length of the strip and a denotes the ultraviolet cutoff (or the lattice spacing). Since S_A is proportional to $\frac{1}{\sqrt{\lambda}}$ in addition to the leading factor N^2 in the planar limit, we cannot explain this result from the free field theory approximation. Therefore we can say that this system is a more interacting theory than the $\mathcal{N}=4$ Yang-Mills, where we can qualitatively reproduce the supergravity result of S_A from the free Yang-Mills [53].

3. Penrose limit of type IIA on $AdS_4 \times CP^3$

3.1 Penrose limit and plane wave solution

We would like to take the Penrose limit [48] of type IIA background $AdS_4 \times CP^3$. We express the metric of AdS_4 by

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh\rho^2 d\Omega_2^2. \tag{3.1}$$

The metric of CP^3 is given by (2.11). We are focusing on the null geodesic defined by

$$\rho = 0, \quad \theta_1 = \theta_2 = 0, \quad \xi = \frac{\pi}{4}. \tag{3.2}$$

We introduce a new angular coordinate

$$\tilde{\psi} = \psi + \frac{\varphi_1 - \varphi_2}{2}. \tag{3.3}$$

The Penrose limit is defined by the following coordinate transformation

$$\frac{t+\tilde{\psi}}{2} = x^+, \quad \tilde{R}^2 \frac{t-\tilde{\psi}}{2} = x^-, \quad \rho = \frac{r}{\tilde{R}}, \quad \theta_i = \frac{\sqrt{2} y_i}{\tilde{R}}, \quad \xi = \frac{\pi}{4} + \frac{y_3}{2\tilde{R}}, \tag{3.4}$$

setting $\tilde{R}$ to infinity with $x^\pm, r, y_1, y_2$ and y_3 kept finite.

In the end we find the following metric in this limit $\tilde{R} \to \infty$

$$\begin{aligned} ds^2_{\text{IIA}} &= -4dx^+dx^- - (r^2 + y_3^2)(dx^+)^2 + dx^+(-y_1^2 d\varphi_1 + y_2^2 d\varphi_2) \\ &\quad + dr^2 + r^2 d\Omega_3^2 + (dy_1^2 + y_1^2 d\varphi_1^2) + (dy_2^2 + y_2^2 d\varphi_2^2) + dy_3^2. \end{aligned} \tag{3.5}$$

[2]Here we employed the explicit value $\mathrm{Vol}(CP^3) = \frac{\pi^2}{12}$ of the volume of CP^3 in the coordinate (2.11).

At the same time, the RR fluxes becomes

$$F_{+y_3} = \frac{k}{2\tilde{R}}, \quad F_{+r\Omega_2} = \frac{3k}{2\tilde{R}} r^2. \tag{3.6}$$

If we define

$$\tilde{\varphi}_1 = \varphi_1 - \frac{x^+}{2}, \quad \tilde{\varphi}_2 = \varphi_2 + \frac{x^+}{2}, \tag{3.7}$$

we can rewrite the metric as

$$\begin{aligned} ds^2_{\rm IIA} &= -4dx^+dx^- - \left(r^2 + y_3^2 + \frac{y_1^2 + y_2^2}{4} \right) (dx^+)^2 \\ &\quad +dr^2 + r^2 d\Omega_3^2 + (dy_1^2 + y_1^2 d\tilde{\varphi}_1{}^2) + (dy_2^2 + y_2^2 d\tilde{\varphi}_2^2) + dy_3^2, \end{aligned} \tag{3.8}$$

which is a familiar form of the plane wave.

If we introduce the Cartesian coordinate $(x_1, \cdots, x_8)$ in an obvious way we get

$$ds^2 = -4dx^+dx^- - \left(\sum_{i=1}^{4} x_i^2 + \frac{1}{4} \sum_{i=5}^{8} x_i^2 \right) (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2, \tag{3.9}$$

with

$$F_{+4} = \frac{k}{2\tilde{R}}, \quad F_{+123} = \frac{3}{2} \frac{k}{\tilde{R}}. \tag{3.10}$$

Since the dilaton is expressed as $e^\phi = \frac{2\tilde{R}}{k}$, we can rewrite the values of RR-fluxes as $e^\phi F_{+4} = 1$ and $e^\phi F_{+123} = 3$, which will be useful later.

This plane-wave background (3.10) in IIA string has been known in the literature [49] and has been shown to have 24 supersymmetries as we expect.

3.2 Gauge theory interpretation

It is argued that the type IIA on $AdS_4 \times CP^3$ is dual to the 't Hooft limit of $\mathcal{N} = 6$ superconformal Chern-Simons theory with the level $(k, -k)$ and the gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ in [45]. Since the gauge theory coupling in Chern-Simons theories is proportional to $\frac{1}{k}$, the 't Hooft coupling is identified with $\lambda = \frac{N}{k}$. Thus the 't Hooft limit is defined as the large N limit with $\lambda = \frac{N}{k}$ kept finite. It is natural to expect that our Penrose limit should correspond to a certain limit of this gauge theory.

The ABJM theory consists of the Chern-Simons $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge potentials at level $(k, -k)$ coupled to the four chiral superfields A_1, A_2, B_1 and B_2, whose structure is very similar[3] to the Klebanov-Witten theory [54]. The fields $(A_1, A_2, \bar{B}_1, \bar{B}_2)$ belong to the $(\mathbf{N}, \bar{\mathbf{N}})$ representation under the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge group and they transform as the fundamental representation under the SU(4) R-symmetry of this $\mathcal{N} = 6$ Chern-Simons theory.

First we relate the transverse scalars in the directions of (X^1, X^2, X^3, X^4) in (2.6) with the scalar fields[4] $(A_1, A_2, \bar{B}_1, \bar{B}_2)$ in the ABJM theory, following the SU(4) R-symmetry.

[3] The Penrose limit of the Klebanov-Witten theory $AdS_5 \times T^{1,1}$ has been studied in [55].

[4] In this paper we also express the scalar field part of the chiral superfield A_i and B_i.

We denote the conformal dimension Δ and define U(1) parts of R-charges J_1, J_2, J_3 as follows (here we still did not perform the shift $\varphi_i \to \tilde{\varphi}_i$ in (3.7))

$$
\begin{aligned}
J_1 &= -i\frac{\partial}{\partial\varphi_1}\Big|_{\tilde{\psi}} = -i\left(\frac{\partial}{\partial\varphi_1} - \frac{1}{2}\frac{\partial}{\partial\psi}\right), \\
J_2 &= -i\frac{\partial}{\partial\varphi_2}\Big|_{\tilde{\psi}} = -i\left(\frac{\partial}{\partial\varphi_2} + \frac{1}{2}\frac{\partial}{\partial\psi}\right), \\
J_3 &= -i\frac{\partial}{\partial\psi}.
\end{aligned}
\tag{3.11}
$$

Notice that in the final forms of three R-charges we fixed $\tilde{\psi}, \varphi_1$ and φ_2 to be constant. Using the dependence of the angles in (2.6) we find

$$
\begin{aligned}
&J_1(A_1) = \frac{1}{4}, \quad && J_1(A_2) = -\frac{3}{4}, \quad && J_1(B_1) = -\frac{1}{4}, \quad && J_1(B_2) = -\frac{1}{4}, \\
&J_2(A_1) = \frac{1}{4}, && J_2(A_2) = \frac{1}{4}, && J_2(B_1) = -\frac{1}{4}, && J_2(B_2) = \frac{3}{4}, \\
&J_3(A_1) = \frac{1}{2}, && J_3(A_2) = \frac{1}{2}, && J_3(B_1) = \frac{1}{2}, && J_3(B_2) = \frac{1}{2}.
\end{aligned}
\tag{3.12}
$$

Now we would like to relate the light-cone momenta p^+ and p^- in the type IIA string to the gauge theoretic quantities assuming the AdS_4/CFT_3 duality. To do this we need to rewrite the metric in terms of $\tilde{\varphi}_i$ instead of φ_i as in (3.8). In this process, we regard any derivative as the one with $\tilde{\psi}, \tilde{\varphi}_1$ and $\tilde{\varphi}_2$ fixed to be a constant. In the end, we find

$$
2p^- = i\frac{\partial}{\partial x^+} = \Delta - J, \qquad 2p^+ = i\frac{\partial}{\partial x^-} = \frac{\Delta + J}{\tilde{R}^2}, \tag{3.13}
$$

where J is defined by

$$
J = J_3 + \frac{1}{2}J_1 - \frac{1}{2}J_2. \tag{3.14}
$$

Explicitly, we get

$$
J(A_1) = \frac{1}{2}, \quad J(A_2) = 0, \quad J(B_1) = \frac{1}{2}, \quad J(B_2) = 0. \tag{3.15}
$$

3.3 World-sheet analysis

First we analyze the bosonic sector. The world-sheet action in the light-cone gauge $X^+ = 2p^+\tau$ looks like (notice $0 \le \sigma \le \pi$)

$$
\begin{aligned}
S_B &= \frac{1}{4\pi\alpha'}\int d\sigma d\tau \partial_a X^\mu \partial_a X^\nu g_{\mu\nu}(X), \\
&= \frac{1}{4\pi\alpha'}\int d\sigma d\tau\left[\sum_{i=1}^{8}\left((\partial_\tau X^i)^2 - (\partial_\sigma X^i)^2\right) - 4(p^+)^2\sum_{i=1}^{4}(X^i)^2 - (p^+)^2\sum_{i=5}^{8}(X^i)^2\right].
\end{aligned}
\tag{3.16}
$$

Then we easily find that the spectrum is given by (setting $\alpha' = 1$)

$$
2p_B^- = \sum_{n=-\infty}^{\infty} N_n^{(1)}\sqrt{1 + \frac{n^2}{(p^+)^2}} + \sum_{n=-\infty}^{\infty} N_n^{(2)}\sqrt{\frac{1}{4} + \frac{n^2}{(p^+)^2}}, \tag{3.17}
$$

where $N^{(1)}$ (and $N^{(2)}$) denote the total occupation number of n-th string modes with respect to the oscillators $\alpha_n^{1,2,3,4}$ (and $\alpha_n^{5,6,7,8}$). We always need to impose the level matching condition

$$\sum_{n=-\infty}^{\infty} n(N_n^{(1)} + N_n^{(2)}) = 0. \tag{3.18}$$

Using the relation $p^+ = \frac{\Delta+J}{2\tilde{R}^2} \simeq \frac{J}{\tilde{R}^2}$, we can rewrite the above formula in term of the gauge theory quantities

$$\Delta - J = \sum_{n=-\infty}^{\infty} N_n^{(1)} \sqrt{1 + \frac{2\pi^2 n^2}{J^2} \cdot \frac{N}{k}} + \sum_{n=-\infty}^{\infty} N_n^{(2)} \sqrt{\frac{1}{4} + \frac{2\pi^2 n^2}{J^2} \cdot \frac{N}{k}}. \tag{3.19}$$

As in the BMN case [48], we expect that the insertion of the string oscillators corresponds to that of the impurity operators in $\mathrm{Tr}(A_1B_1)^J$. Indeed, A_1B_1 and their powers are the unique operators which satisfy[5] $\Delta - J = 0$, as is clear from (3.15). Also notice that $\mathrm{Tr}(A_1B_1)^J$ is the chiral primary operator. By inspecting the R-charge of impurities we can easily identify (assuming the zero mode $n = 0$) the 4 oscillators $\alpha_0^{5,6,7,8}$ with

$$A_1B_2, \quad A_1\bar{A}_2, \quad A_2B_1, \quad ,\bar{B}_2B_1. \tag{3.20}$$

Indeed these four operators satisfy $\Delta - J = \frac{1}{2}$. Therefore we argue that the oscillators $(\alpha_0^5 - i\alpha_0^6, \alpha_0^5 + i\alpha_0^6, \alpha_0^7 - i\alpha_0^8, \alpha_0^7 + i\alpha_0^8)$ are dual to the replacement procedures

$$A_1 \to A_2, \quad B_1 \to \bar{A}_2, \quad A_1 \to \bar{B}_2, \quad B_1 \to B_2. \tag{3.21}$$

On the other hand, we expect that the three oscillators $\alpha_0^{1,2,3}$ should be dual to the covariant derivative D_μ, where $\mu = 0, 1, 2$. We still need to identify one more. There are six other operators which satisfy $\Delta - J = 1$:

$$A_1\bar{A}_1, \quad \bar{A}_2\bar{B}_2, \quad A_2\bar{A}_2, \quad \bar{B}_1B_1, \quad \bar{B}_2B_2, \quad \bar{B}_2\bar{A}_2. \tag{3.22}$$

Among them only $A_1\bar{A}_1$ and $\bar{B}_2B_2$ are independent from the double excitations of the previous operations in (3.20). Therefore, α_0^4 is expected to be dual to a linear combination of these operators.

Next we turn to the fermionic sector (we follow the convention in [56]). The fermion part in the light cone gauge $\Gamma^+ S = 0$ in the Green-Schwartz formalism looks like

$$S_F = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \partial_a X^\mu \bar{S} \Gamma_\mu (\delta^{ab} - \epsilon^{ab}\Gamma^{11}) D_b S, \tag{3.23}$$

where S is a ten dimensional Majorana spinor and the covariant derivative D_b is the pullback to the world-sheet of the supercovariant derivative in IIA supergravity

$$D_\mu \epsilon = \nabla_\mu \epsilon + \frac{e^\phi}{4} F_{\mu\nu}\Gamma^\nu\Gamma^{11}\epsilon - \frac{e^\phi}{(4!)^2}(3F_{\alpha\beta\gamma\delta}\Gamma^{\alpha\beta\gamma\delta}\Gamma_\mu - F_{\alpha\beta\gamma\delta}\Gamma_\mu\Gamma^{\alpha\beta\gamma\delta})\epsilon. \tag{3.24}$$

[5] Here we neglect the contribution from the operators with Wilson line attached [45] since we are assuming k is large.

Then the action is simplified up to a constant

$$S_F = \int d\tau d\sigma \left[\bar{S}\Gamma_+(\partial_\tau + \Gamma^{11}\partial_\sigma)S + \frac{p^+}{2}\bar{S}\Gamma_+(\Gamma^4\Gamma^{11} + 3\Gamma^{123})S\right]. \tag{3.25}$$

The equation of motion becomes

$$(\partial_\tau + \Gamma^{11}\partial_\sigma)S = -\frac{p^+}{2}(\Gamma^4\Gamma^{11} + 3\Gamma^{123})S. \tag{3.26}$$

By multiplying $\partial_\tau - \Gamma^{11}\partial_\sigma$ we obtain

$$(\partial_\tau^2 - \partial_\sigma^2)S = (2p^+)^2\mu^2 S, \tag{3.27}$$

where

$$\mu^2 \equiv -\left(\frac{\Gamma^4\Gamma^{11} + 3\Gamma^{123}}{4}\right)^2. \tag{3.28}$$

Since $16\mu^2 = 10 - 6\Gamma^{123411}$, we can conclude that among eight physical fermions, half of them have $\mu^2 = 1$, while other half do $\mu^2 = \frac{1}{4}$. This mass spectrum is exactly the same as the bosonic one. Thus we find the fermion spectrum

$$2p_F^- = \sum_{n=-\infty}^{\infty} N_n^{(1)}\sqrt{1 + \frac{n^2}{(p^+)^2}} + \sum_{n=-\infty}^{\infty} N_n^{(2)}\sqrt{\frac{1}{4} + \frac{n^2}{(p^+)^2}}, \tag{3.29}$$

and the values of $\Delta - J$ for fermions are given by the same formula (3.19). Among totally sixteen fermions in the dual gauge theory, four fermions satisfy $\Delta - J = \frac{1}{2}$ and other four fermions do $\Delta - J = \frac{3}{2}$, while the rest eight fermions have $\Delta - J = 1$. Therefore we find that the string spectrum is consistent with this gauge theory fermionic operators at least for the zero modes.

In this way have shown a nice matching between the zero modes of the IIA string theory in the Penrose limit and the gauge theory operators. This is of course expected since the operators dual to zero modes (or KK modes) are protected under the change of the coupling constant.

4. BMN like operators

Motivated by the analysis of Penrose limit in the previous section we would like to examine non-BPS operators in the ABJM theory. Especially, we are interested in the BMN-like operators[6] (almost BPS operators). Indeed it is not difficult to find analogous operators (refer also to [2] for similar operators in less supersymmetric Chern-Simons theory.).

We would like to concentrate on the following operators assuming J is very large

$$\mathcal{O}_n = \frac{1}{\sqrt{2J}}\sum_{l=0}^{J} e^{2\pi i \frac{ln}{J}}\mathrm{Tr}[(A_1B_1)^l A_1B_2(A_1B_1)^{J-l}(A_1B_2)]. \tag{4.1}$$

Notice that $\mathcal{O}_0$ is chiral primary since the index i and j of A_i and B_j are both symmetrized independently. We can treat the impurity of the form A_2B_1 exactly in the same way.

[6]The existence of the spin chain like structure was already suggested in [2, 45].

4.1 Anomalous dimension

We would like to compute two point functions of these operators and obtain the anomalous dimensions to leading order. Since we know that the operator $\mathcal{O}_0$ is chiral primary and its anomalous dimension is vanishing, we have only to consider the Feynman diagrams whose results depend on n. Then the relevant part of the Lagrangian looks like (we follow the convention in [45])

$$\mathcal{L} = \sum_{i=1}^{2} \left(\partial_\mu A^i \partial^\mu \bar{A}^i + \partial_\mu B^i \partial^\mu \bar{B}^i \right) + \frac{16\pi^2}{k^2} \mathrm{Tr}[B_2 A_1 B_1 \bar{B}_2 \bar{A}_1 \bar{B}_1] + \frac{16\pi^2}{k^2} \mathrm{Tr}[B_1 A_1 B_2 \bar{B}_1 \bar{A}_1 \bar{B}_2]. \tag{4.2}$$

Then the propagator is normalized as follows

$$\langle A^i_{a\bar{b}}(x) \bar{A}^j_{\bar{c}d}(0) \rangle = \langle B^i_{\bar{b}a}(x) \bar{B}^j_{d\bar{c}}(0) \rangle = \frac{\delta_{ij}\delta_{da}\delta_{\bar{c}\bar{b}}}{4\pi|x|}, \tag{4.3}$$

where we neglect any interactions which do not affect our leading computation of the anomalous dimension.

Since the two interactions in (4.2) exchange B_2 with the two nearest B_1s, respectively, they produce the phase factors $e^{\pm 2\pi i \frac{n}{J}}$, which is very similar to the BMN analysis [48]. Also notice that the insertion of either of these interactions adds two loops in the fat diagram and leads to N^2 factor. Therefore we obtain

$$\langle \mathcal{O}_n(x) \overline{\mathcal{O}}_n(0) \rangle = \frac{\mathcal{N}}{|x|^{2(J+2)}} \left(1 + \frac{1}{(4\pi)^3} \cdot \frac{32\pi^2 N^2}{k^2} (\cos\frac{2\pi n}{J} - 1) \cdot I(x) \right), \tag{4.4}$$

where $I(x) = |x|^3 \int \frac{dy^3}{|y|^3|x-y|^3}$ and $\mathcal{N}$ is a normalization factor. Since $I(x) \sim 8\pi \log x\Lambda$ (Λ is the cutoff), we can conclude that the leading anomalous dimension δ_n^{CS} of $\mathcal{O}_n$ is given by

$$\delta_n^{\mathrm{CS}} = 4\pi^2 \frac{N^2 n^2}{k^2 J^2} + \cdots, \tag{4.5}$$

where $\cdots$ denotes the higher order terms with respect to $\frac{N}{kJ}$.

It is also useful to remember that in the original analysis of BMN operators there are two different coupling constants: one is the rescaled 't Hooft coupling $\frac{\lambda}{J^2}$ and the other is the effective string coupling $\frac{J^2}{N}$ [57]. The latter appears when we consider the non-planar diagrams, which we neglected in the above. In our Chern-Simons gauge theory, we can see that the non-planar corrections come with the same factor $\frac{J^2}{N}$. Since in our argument which derives (4.5), we keep the rescaled 't Hooft coupling $\frac{N}{kJ}$ a small value, the non-planar correction is negligible if $N \ll k^2$.

4.2 Comparison with IIA plane wave

One may naively guess that the Penrose limit of the IIA string studied in section 3 corresponds to the BMN-like limit assumed in the previous subsection

$$\frac{N}{kJ} = \text{finite}, \qquad N \ll k^2, \tag{4.6}$$

as the analogous relation was true in the celebrated duality between $AdS_5 \times S^5$ and the four dimensional $N = 4$ Yang-Mills theory [48]. However, this does not seem to be the case here, even though about the chiral primary operators there is a nice matching between them as we have seen in the previous section. In fact, the anomalous dimension found in the Penrose limit reads (see (3.19))

$$\delta_n^{\mathrm{IIA}} = \frac{2\pi^2 N n^2}{kJ^2} + \cdots, \tag{4.7}$$

for the impurities of A_1B_2 and A_2B_1. This is different from the result (4.5) obtained from the IIA string spectrum on the plane wave by the factor $\frac{2N}{k}$. In this string theoretic calculation in the Penrose limit, we need to keep the string coupling $e^{2\phi}$ small and p^+ finite, which requires

$$\frac{1}{(p^+)^2} \sim \frac{N}{kJ^2} = \text{finite}, \quad e^{2\phi} \sim \sqrt{\frac{N}{k^5}} \ll 1. \tag{4.8}$$

We would like to argue that the disagreement between the leading anomalous dimensions (4.5) and (4.7) is not a contradiction but is due to the violation of the BMN scaling (a similar phenomenon in other type IIA backgrounds has been pointed out in [50]). Notice that the violation of BMN scaling in this sort of computations (i.e. near BPS states to the leading order of the large J limit) does not occur in the AdS_5/CFT_4 duality of the $\mathcal{N} = 4$ super Yang-Mills theory.

In other words, we expect that the anomalous dimension of $\mathcal{O}_n$ in the large J limit of the ABJM theory is given by

$$\delta_n = f(\lambda)\frac{n^2}{J^2} + \cdots, \tag{4.9}$$

in terms of a certain function $f(\lambda)$ of $\lambda = \frac{N}{k}$. Our results (4.5) and (4.7) predict the following behaviors[7]

$$f(\lambda) \to 2\pi^2\lambda \quad (\lambda \to \infty), \qquad \text{and} \qquad f(\lambda) \to 4\pi^2\lambda^2 \quad (\lambda \to 0). \tag{4.10}$$

It will be very interesting to compute the function $f(\lambda)$ exactly from the Chern-Simons theory.

5. Free $\mathcal{N} = 6$ Chern-Simons theory on $S^1 \times S^2$

Obviously, another limit which we can take to make a given theory simpler and more tractable is the weak coupling limit. We would like to finish this paper by studying the weak coupling limit $k \to \infty$ of the $\mathcal{N} = 6$ Chern-Simons theory on $S^1 \times S^2$. We will show that the Hagedorn/deconfinement transition will occur in almost the same way as in the $\mathcal{N} = 4$ free Yang-Mills on $S^1 \times S^3$ [58, 59].

[7] It is not difficult to find functions with these properties. Indeed, we can consider functions like $f(\lambda) = 4\pi^2\frac{\lambda^2}{1+2\lambda}$ or $f(\lambda) = 4\pi^2\frac{\lambda^2}{\sqrt{1+4\lambda^2}}$, for example.

The original ABJM action in this limit becomes

$$S_{\rm ABJM} = \int d^3x \frac{1}{4\pi} {\rm Tr}\left[\left(A_{(1)} \wedge dA_{(1)} + \frac{2}{3\sqrt{k}} A_{(1)}^3\right) - \left(A_{(2)} \wedge dA_{(2)} + \frac{2}{3\sqrt{k}} A_{(2)}^3\right)\right]$$
$$+ {\rm Tr} \sum_{i=1,2} \left[|D_\mu^{(+)} A_i|^2 + |D_\mu^{(-)} B_i|^2 + i\bar{\psi}_i \not{D}^{(+)}\psi_i + i\bar{\chi}_i \not{D}^{(-)}\chi_i\right] + \mathcal{O}\left(\frac{1}{k}\right), \quad (5.1)$$

where we define the covariant derivatives

$$D_\mu^{(\pm)} = \nabla_\mu \pm \frac{i}{\sqrt{k}}(A_{(1)\,\mu} \otimes \mathbf{1} - \mathbf{1} \otimes A_{(2)\,\mu}). \quad (5.2)$$

We must be careful in taking k infinity since a naive treatment spoils the Gauss' law constraint [59]. We must choose the gauge fixing by the temporal gauge

$$A_{(a)\,0}(x) = \sqrt{k}\, a_{(a)}, \quad (a = 1, 2). \quad (5.3)$$

Under this gauge fixing, the action (5.1) on $S^1 \times S^2$ becomes

$$S_{\rm free} = iS_{\rm CS}(A_{(1)}; S^1 \times S^2) - iS_{\rm CS}(A_{(2)}; S^1 \times S^2) \quad (5.4)$$
$$+{\rm Tr}\sum_{i=1,2} \left[\bar{A}_i\left(-(D_\mu'^{(+)})^2 + \frac{\mathcal{R}}{8}\right)A_i + \bar{B}_i\left(-(D_\mu'^{(-)})^2 + \frac{\mathcal{R}}{8}\right)B_i + i\bar{\psi}_i \not{D}'^{(+)}\psi_i + i\bar{\chi}_i \not{D}'^{(-)}\chi_i\right],$$

where we included $\frac{\mathcal{R}}{8}$ term which arises from a conformal coupling of the scalar field and defined $D_\mu'^{(\pm)} = (D_0'^{(\pm)} \equiv \partial_0 \pm i(a_{(1)} \otimes \mathbf{1} - \mathbf{1} \otimes a_{(2)}), \nabla_1, \nabla_2)$. The Ricci scalar is $\mathcal{R} = 2$ for the unit two sphere, and integrating the matter fields out gives the one-loop effective action

$$\begin{aligned} {\rm Tr}\ln\left(-(D_0'^{(\pm)})^2 - \nabla^2 + \frac{\mathcal{R}}{8}\right) &= -\sum_{n=1}^\infty \frac{1}{n} z_B(x^n)\,({\rm tr}U^n\, {\rm tr}V^{-n} + {\rm tr}U^{-n}\,{\rm tr}V^n), \\ {\rm Tr}\ln(-(\not{D}'^{(\pm)})^2) = Tr\ln\left(-(D_0'^{(\pm)})^2 - \nabla^2 + \frac{\mathcal{R}}{8}\right) &= \sum_{n=1}^\infty \frac{(-)^{n+1}}{n} z_F(x^n)\,({\rm tr}U^n\,{\rm tr}V^{-n} + {\rm tr}U^{-n}\,{\rm tr}V^n), \end{aligned} \quad (5.5)$$

where we denote $x = e^{-\beta}$ and introduce $U = e^{i\beta\alpha_{(1)}}$, $V = e^{i\beta\alpha_{(2)}}$ as Wilson loops along S^1. We omit the irrelevant terms independent of α and define the single-particle partition function of bosons and fermions as

$$z_B(x) = \frac{x^{\frac{1}{2}}(1+x)}{(1-x)^2}, \qquad z_F(x) = \frac{2x}{(1-x)^2}. \quad (5.6)$$

After all, the partition function becomes the expectation value of the Wilson loops of $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons gauge theory

$$\begin{aligned} Z = &\int [DA_{(1)}][DA_{(2)}] \exp\Big[iS_{\rm CS}(A_{(1)}; S^1 \times S^2) - iS_{\rm CS}(A_{(2)}; S^1 \times S^2) \\ &+ \sum_{n=1}^\infty \frac{1}{n}(4z_B(x^n) + (-)^{n+1} 4z_F(x^n))({\rm tr}U^n\,{\rm tr}V^{-n} + {\rm tr}U^{-n}\,{\rm tr}V^n)\Big], \\ = &\left\langle \exp\left[\sum_{n=1}^\infty \frac{1}{n} z_n(x)({\rm tr}U^n\,{\rm tr}V^{-n} + {\rm tr}U^{-n}\,{\rm tr}V^n)\right]\right\rangle_{S^1\times S^2}, \end{aligned} \quad (5.7)$$

where, in the second equality, we define $z_n(x) = 4z_B(x^n) + (-)^{n+1}4z_F(x^n)$. It is known that only singlet representation of the Wilson loops takes non-zero expectation value in Chern-Simons gauge theory on $S^1 \times S^2$ [60], then we can rewrite the above expression as the matrix model

$$Z = \int [dU][dV] \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} z_n(x)(\mathrm{tr}U^n\, \mathrm{tr}V^{-n} + \mathrm{tr}U^{-n}\, \mathrm{tr}V^n)\right]. \tag{5.8}$$

Once taking the large-N limit, we can obtain the effective action

$$I_{\mathrm{eff}} = N^2 \sum_{n=1}^{\infty} \frac{1}{n} \left(|u_n|^2 + |v_n|^2 - z_n(x)(u_n v_{-n} + u_{-n} v_n)\right), \tag{5.9}$$

where $u_n \equiv \mathrm{tr}U^n/N$, $v_n \equiv \mathrm{tr}V^n/N$ and the first two terms in the right hand side come from the measure.

We now consider the saddle point of the matrix model action (5.9). The eigenvalues λ of the quadratic form in (5.9) with respect to (u_n, v_n), read $\lambda = 1 \pm z_n(x)$. Thus the trivial saddle point $u_n = v_n = 0$ is dominated if $z_1(x) < 1$ since $z_n(x)$ is monotonically decreasing function of n. For $z_1(x) > 1$, one of the eigenvalues becomes negative and the action is dominated by another saddle point which gives order N^2 free energy. Then, there is a deconfinement transition at $z_1(x) = 1$ and the Hagedorn temperature is calculated using (5.6) as $T_H = \frac{1}{\log(17+12\sqrt{2})} \sim 0.283648$.

In this way we have shown that in the large k limit (free limit), a Hagedorn/deconfinement transition occurs in the ABJM theory. In the strong coupling region $\frac{N}{k} \gg 1$, this is expected from supergravities [61]: both the IIA string on CP^3 and the M-theory on S^7/Z_k have the AdS_4 black hole solution. To understand the finite k region in the gauge theory side, which is dual to the M-theory, we need to take the non-singlet flux contributions [45] into account and this will be an interesting future problem.

6. Conclusion

In this paper we examined the Penrose limit of the type IIA string on $AdS_4 \times CP^3$, which is argued to be dual to the $\mathcal{N} = 6$ Chern-Simons gauge theory (ABJM theory) in the 't Hooft limit. We obtained the resulting plane wave background and compute the string spectrum in terms of gauge theoretic quantities. For BPS operators, we find the agreement between the IIA string and the ABJM theory. Also the string spectrum in the plane wave limit provides us with an important prediction of the anomalous dimensions in certain sectors which satisfy $J \sim \sqrt{\frac{N}{k}}$ in the ABJM theory. We also analyzed the gauge theory sides and argued that we can define BMN-like (almost BPS) operators when the R-charge J is large. We calculated the leading anomalous dimensions for these BMN-like operators and found that the results are different from the ones computed in the IIA string on the plane wave. This shows that the BMN scaling in the ABJM theory is violated already in this near BPS sector as opposed to the $\mathcal{N} = 4$ super Yang-Mills theory. This issue definitely deserves future studies.

We also examined the weak coupling limit $k \to \infty$ of the ABJM theory on $S^1 \times S^2$ and evaluated the partition function at finite temperature. We showed that the Hagedorn/deconfinement transition occurs in this limit of the ABJM theory as naturally expected.

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